Problem of the Week #6

(Spring 2024)

I arrived at the Magic: the Gathering tournament with my deck of 24 land cards and 36 other cards. On average, once I shuffle the deck (but before I start to draw), how many places will there be in the deck where a land card touches a non-land card (*i.e.*, comes immediately before or after it in the deck)?

Solution:

Proof. Define the random variable X_i so that $X_i = 1$ if exactly one of the cards i and $i+1$ is a land, and $X_i = 0$ otherwise. The expected value of X_i is the probability that exactly one of i and $i+1$ are lands, which is

$$E(X_i) = \frac{24}{60} \cdot \frac{36}{59} + \frac{36}{60} \cdot \frac{24}{59} = \frac{2 \cdot 36 \cdot 24}{60 \cdot 59} = \frac{6 \cdot 24}{5 \cdot 59}.$$

By the linearity of expectation, the expected number of places where land cards touch non-land cards is

$$E\left(\sum_{i=1}^{59} X_i\right) = \sum_{i=1}^{59} E(X_i) = 59 \cdot \frac{6 \cdot 24}{5 \cdot 59} = \frac{144}{5} = 28.8.$$

$\square$

Source: Frederick Mosteller, "The Theater Row," *Fifty Challenging Problems in Probability With Solutions*, Dover Publications (1965), 29–30.