UNIVERSITY OF WISCONSIN
PLATTEVILLE
DEPARTMENT OF MATHEMATICS

Problem of the Week #3

(Spring 2024)

Given a positive integer k, what is the largest value of n for which $\dfrac{n!-k}{n-k}$ is an integer?

Solution:

The fraction $\dfrac{n!-k}{n-k}$ is an integer when $\boxed{n=2k}$, but not for any greater value of n.

Proof. First, $\dfrac{(2k)!-k}{2k-k} = 2(2k-1)!-1$, which is an integer.

Now suppose $n > 2k$. Then $n > n-k > k$, so $n!$ is divisible by $n-k$, but k is not, and so $n!-k$ is not, either. □

Source: Matthew Scroggs, "Advent calendar 2023," `https://www.mscroggs.co.uk/puzzles/advent2023/4`.