



PROBLEM OF THE WEEK #1
(Spring 2024)

Suppose that $f(x)$ satisfies the identity $2f(x) - f(-1/x) = x$ for all $x \neq 0$. Find an explicit formula for $f(x)$.

Solution:

Solution. By substituting $-1/x$ for x in the given identity, we have

$$\begin{cases} 2f(x) - f(-1/x) = x, \\ 2f(-1/x) - f(x) = -1/x. \end{cases}$$

Adding twice the first equation to the second, we get $3f(x) = 2x - \frac{1}{x}$, so

$$f(x) = \frac{2}{3}x - \frac{1}{3x} = \boxed{\frac{2x^2 - 1}{3x}}.$$

□

Source: Elliott Line, “actually good math problems,” [facebook.com](https://www.facebook.com/elliottline), 26 November 2023.