PROBLEM OF THE WEEK #8

(Spring 2022)

Fill in the squares using each digit from 1 through 9 exactly once to make a true equation:

$$\frac{\square}{\square \times \square} + \frac{\square}{\square \times \square} + \frac{\square}{\square \times \square} = 1$$

Solution:

$$\frac{1}{3 \times 6} + \frac{5}{8 \times 9} + \frac{7}{2 \times 4} = \frac{4 + 5 + 63}{72} = 1.$$

Remark. You can speed up your search for a solution by noticing that 5 and 7 have to be numerators. If 5 appears in the first denominator (for example), then the other two fractions have denominators that are not multiples of 5. After we find the least common denominator, the second and third fractions have multiples of 5 for numerators. Since the three fractions add up to 1, the three numerators add up to a multiple of 5, so the first numerator is also a multiple of 5 — a contradiction.

Source: Yoshigahara, Nobuyuki. "Puzzle 27: Pandigital (KOMACHI) Fraction II." *Puzzles 101: A Puzzlemaster's Challenge.* CRC Press (2004), pp. 18, 80.