



PROBLEM OF THE WEEK #6  
(Spring 2022)

After the Math Department's *Magic: the Gathering* tournament, the five competitors told their colleagues how it turned out. But, mathematicians being what they are, each competitor told only a half-truth: that is, one true and one untrue statement. Here's what they said:

Allen: Deis came in second. I finished third.

Black: I won the whole thing, and Chang finished second.

Chang: I got third place; poor Black was last.

Deis: I was in second place, and Frayer was fourth.

Frayer: I ended up fourth. Allen was the champion!

What were the true results? [Assume there were no ties.]

**Solution:**

In order, the finishers were Deis, Chang, Allen, Frayer, and Black.

*Proof.* It's easy to check that this placement gives each speaker exactly one true statement. But could there be another solution?

Suppose for contradiction that Frayer didn't finish fourth. Then Deis and Frayer lied about that, so Deis finished second and Allen won. So Black lied about winning, which means Chang finished second. But Deis and Chang couldn't both finish second. Hence Frayer did finish fourth. Since Deis told the truth about that, Deis wasn't second. Allen lied about that, so Allen was third. Chang lied about finishing third, so Black was fifth. Black lied about his finish, so Chang was second. By elimination, Deis won the tournament.  $\square$

**Source:** Grätzer, George. "Half-Truths." *Train Your Brain: A Year's Worth of Puzzles*. CRC Press (2011), pp. 53, 182-3.