UNIVERSITY OF WISCONSIN
PLATTEVILLE
DEPARTMENT OF MATHEMATICS

# Problem of the Week #4

(Spring 2022)

Let $x$ and $y$ be real numbers such that

$$\begin{cases} \log \sin x + \log \cos x = -1, \\ \log(\sin x + \cos x) = -1 + \frac{1}{2} \log y, \end{cases}$$

where "log" denotes the common (base-10) logarithm. Solve for $y$.

**Solution:**

The first equation tells us that $\log(\sin x \cos x) = -1$, so $\sin x \cos x = \frac{1}{10}$. Therefore:

$$\begin{aligned} \log y &= 2 + 2\log(\sin x + \cos x) \\ &= 2 + \log[(\sin x + \cos x)^2] \\ &= 2 + \log(\sin^2 x + 2\sin x \cos x + \cos^2 x) \\ &= \log 100 + \log\left(1 + 2 \cdot \frac{1}{10}\right) \\ &= \log\left(100 \cdot \frac{12}{10}\right) \\ &= \log 120. \end{aligned}$$

Hence $\boxed{y = 120}$.

**Source:** Adapted from Problem #4 of the 2003 American Invitational Mathematics Exam. In: Scott A. Annin. *A Gentle Introduction to the American Invitational Mathematics Exam.* The Mathematical Association of America (2015), 120-121.