PROBLEM OF THE WEEK #3
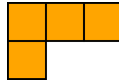(Spring 2022)

The L-block in Tetris is made of four 1×1 squares joined along their edges.
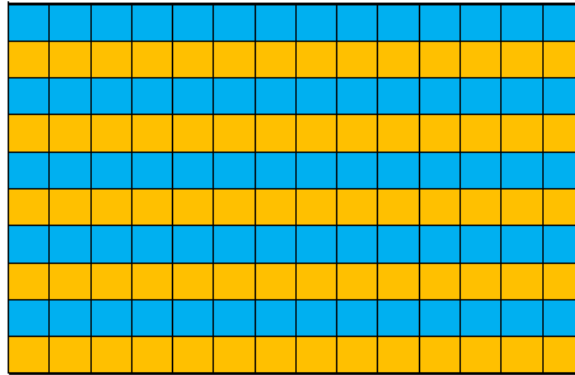
Can an 18×10 rectangle be filled with non-overlapping L-blocks?

Solution:

No.

Proof.

Color the 18×10 rectangle so that it has 5 blue rows alternating with 5 orange rows. No matter how you place an L-block, it will cover three squares of one color and one square of the other color.

Suppose for the sake of contradiction that the 18×10 rectangle has been tiled with L-blocks. Let B be the number of L-blocks that cover 3 blue squares, and R be the number that cover 3 orange squares. Then $\begin{cases} 3B + R &= 90, \\ B + 3R &= 90. \end{cases}$ After solving this system, we have $B = R = \frac{45}{2}$, which is impossible. $\qquad \square$

Source: David Klarner. In: Ross Honsberger. *Mathematical Gems II.* The Mathematical Association of America (1976), 63-65.