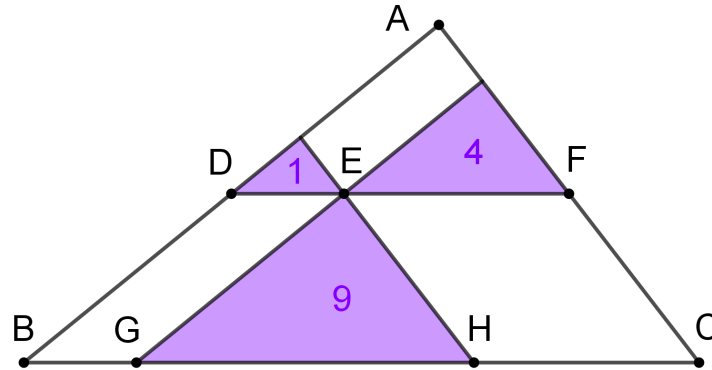# PROBLEM OF THE WEEK #6

(Spring 2021)

Lines parallel to the sides of a triangle T intersect at a single point inside T, forming three smaller triangles of area 1, 4, and 9. Find the area of T.

Solution:
The area of T is 36.

Proof. Name points as shown. Each shaded triangle is similar to T, and since their areas are in the ratio $1 : 4 : 9$, their sides are in the ratio $1 : 2 : 3$. Thus $DE + EF = GH$. On the other hand, $DE = BG$ and $EF = HC$, so

$$BC = BG + GH + HC = DE + (DE + EF) + EF = 2(DE + EF).$$

That is, the base of T is twice the base of $\triangle EGH$. By similarity, the altitude of T is twice the altitude of $\triangle EGH$, and thus the area of T is $4 \cdot 9 = 36$. $\qquad \square$

Source: Titu Andreescu and Jonathan Kane. *Purple Comet! Math Meet: The first ten years.* Plano: XYZ Press, LLC (2013), 99–100.