University of Wisconsin
PLATTEVILLE
Department of Mathematics

Problem of the Week #4

(Spring 2020)

Give a trigonometry-free description of the triples (A, B, C) for which

$$\tan A + \tan B + \tan C = \tan A \tan B \tan C.$$

Solution:

The equation holds if and only if $A + B + C$ is an integer multiple of π.

Proof.

$$\begin{aligned}
\tan A + \tan B + \tan C &= \tan A \tan B \tan C \\
\tan C - \tan A \tan B \tan C &= -\tan A - \tan B \\
\tan C(1 - \tan A \tan B) &= -(\tan A + \tan B) \\
\tan C &= -\frac{\tan A + \tan B}{1 - \tan A \tan B} \\
\tan C &= -\tan(A + B) \\
\tan C &= \tan(-A - B) \\
C &= -A - B + n\pi \qquad (n \in \mathbb{Z}) \\
A + B + C &= n\pi \qquad (n \in \mathbb{Z})
\end{aligned}$$

□

Source: Benjamin L. Schwartz. "Q519." *Mathematics Magazine* **44**:3 (May 1971), p. 165.