UNIVERSITY OF WISCONSIN
PLATTEVILLE
DEPARTMENT OF MATHEMATICS

Problem of the Week #6

(Spring 2019)

The decimal form of the integer N consists of 2019 nines in a row: $N = \underbrace{999\cdots999}_{2019}$.

How many nines occur in the decimal form of N^3?

Solution:
There are 4037 nines in N^3.

Proof. Suppose, more generally, that N consists of k nines in a row.* Then for $k > 1$:

$$\begin{aligned} N &= 10^k - 1 \\ N^3 &= 10^{3k} - 3 \cdot 10^{2k} + 3 \cdot 10^k - 1 \\ &= 10^{3k} + (-10 + 7) \cdot 10^{2k} + 3 \cdot 10^k - 1 \\ &= 10^{3k} - 10^{2k+1} + 7 \cdot 10^{2k} + 3 \cdot 10^k - 1 \\ &= 10^{2k+1}(10^{k-1} - 1) + 7 \cdot 10^{2k} + 3 \cdot 10^k - 1 \\ &= \underbrace{99\cdots99}_{k-1}\,7\,\underbrace{00\cdots00}_{k-1}\,2\,\underbrace{99\cdots99}_{k}. \end{aligned}$$

(For example, $999^3 = 997002999$.) Thus N^3 contains $2k - 1$ nines, $k - 1$ zeros, a seven, and a two. When $k = 2019$, there are $2(2019) - 1 = 4037$ nines in N^3. □

Source: Adapted from the 2009 Dutch Mathematical Olympiad, reprinted in Van den Brandhof, Alex, *et al.*, eds. *Half a Century of Pythagoras Magazine*. The Mathematical Association of America (2015), 204.

*A pleasant assumption for all dog lovers and dentists.