



PROBLEM OF THE WEEK #4
(Spring 2019)

Solve:

$$\sqrt{6 - \sqrt{6 + x}} = x$$

Solution:

Let $y = \sqrt{6 + x}$. This means that $x = \sqrt{6 - y}$. Notice that $x > 0$ and $y > 0$. We obtain:

$$\begin{cases} 6 + x = y^2 \\ 6 - y = x^2 \end{cases}$$

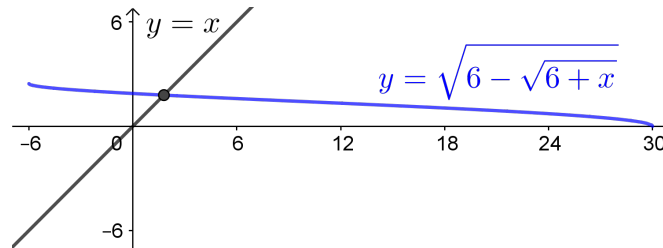
Subtracting these equations, we get:

$$\begin{aligned} x + y &= y^2 - x^2 \\ x^2 - y^2 + x + y &= 0 \\ (x + y)(x - y + 1) &= 0 \end{aligned}$$

Since $x + y > 0$, we have $x - y + 1 = 0$, which means $y = x + 1$. We back-substitute:

$$\begin{aligned} 6 - x - 1 &= x^2 \\ 0 &= x^2 + x - 5 \\ x &= \frac{-1 \pm \sqrt{1 + 4 \cdot 5}}{2} \end{aligned}$$

Since $x > 0$, we have $x = \frac{-1 + \sqrt{21}}{2}$.



Source:

- [1] D. O. Shklarsky, N. N. Chentzov, and I. M. Yaglom, *Problem 168*, The USSR Olympiad Problem Book: Selected Problems and Theorems of Elementary Mathematics, 3rd ed. (Irving Sussman, ed.), translated by John Maykovich, Dover Publications, Inc., New York, 1993, pp. 39, 266.