University of Wisconsin
PLATTEVILLE
DEPARTMENT OF MATHEMATICS

Problem of the Week #2

(Spring 2019)

Recently, some of us attended a presentation in which it was mentioned that in 2016, Peter Trueb had computed the value of π to 22,459,157,718,361 decimal places, and an audience member asked if he had printed them all out.

Suppose that Trueb had in fact printed them all out, and stored the results in stacks on a concrete slab. How large would the slab have to be, assuming that you don't want the concrete to crack?

[To solve this problem, you will need to use information that is not given here. State your assumptions!]

Solution:

Answers may vary. Here's some relevant data:

- In 12-point font, with 1″ margins, 3600 characters fit on one side of one letter-size sheet of paper.
- One hundred sheets of typical ("20 lb") letter-size paper weigh 1 lb.
- The compressive strength of concrete is usually specified at $3000\,\frac{\text{lb}}{\text{in}^2}$.

Thus, if we print on both sides, the area of the slab should be at least

$$22{,}459{,}157{,}718{,}361\,\text{digits}\cdot\frac{1}{7200}\,\frac{\text{sheet}}{\text{digit}}\cdot\frac{1}{100}\,\frac{\text{lb}}{\text{sheet}}\cdot\frac{1}{3000}\,\frac{\text{in}^2}{\text{lb}}\cdot\frac{1}{144}\,\frac{\text{ft}^2}{\text{in}^2}\approx\boxed{72.2\,\text{ft}^2}.$$

On the other hand. . . at most 111 stacks of letter-size paper could be stored on a slab of that area. Since a 1″ stack of paper contains about 300 sheets, each stack's height is at least

$$22{,}459{,}157{,}718{,}361\,\frac{\text{digits}}{\text{slab}}\cdot\frac{1}{7200}\,\frac{\text{sheet}}{\text{digit}}\cdot\frac{1}{111}\,\frac{\text{slab}}{\text{stack}}\cdot\frac{1}{300}\,\frac{\text{in}}{\text{sheet}}\cdot\frac{1}{12}\,\frac{\text{ft}}{\text{in}}\approx 7806\,\frac{\text{ft}}{\text{stack}}$$

ignoring any extra compression at the bottom of each very heavy stack. This may suggest either that the slab should be larger, so that the stacks won't have to be so very tall, or that the slab can be made slightly smaller due to the decreasing effect of gravity at high altitude. If we choose, for example, to limit the stacks to 2722 ft (the height of the Burj Khalifa, the tallest structure ever built by humans), we will require a slab of area $72.2\cdot\frac{7806}{2722}\approx 207\,\text{ft}^2$.

Source:

[1] James Swenson, *Page of Digits*, available at `https://people.uwplatt.edu/~swensonj/PageOfDigits.pdf`.

[2] Pieter Vanderwerf, *Concrete floor slabs*, available at `https://www.concreteconstruction.net/how-to/construction/concrete-floor-slabs_o`.

[3] Jackie Vlahos, *Paper Weight Chart — Thickness of Paper Explained*, available at `https://www.printi.com/blog/thickness-of-paper/`.

[4] Wikipedia, *Chronology of computation of π*, available at `https://en.wikipedia.org/wiki/Chronology\_of\_computation\_of\_$\pi$`.

[5] ______, *List of tallest structures*, available at `https://en.wikipedia.org/wiki/List\_of\_tallest\_structures`.