University of Wisconsin
PLATTEVILLE
DEPARTMENT OF MATHEMATICS

Problem of the Week #8

(Spring 2017)

The NCAA men's basketball tournament features 64 teams* in a 6-round single-elimination tournament. In each round, every team plays one game, and the losing teams are immediately eliminated from the tournament.

This year, I entered a bracket contest: before the tournament started, I submitted predictions of the winners of all the games. Unfortunately, I hadn't been paying much attention to college basketball, so I made all of my predictions by tossing coins.

According to the contest rules, every correct guess in the first round earns me one point; this value doubles every round, so that (for example) guessing the winner of the championship game is worth 32 points. [But it's hard to earn those points, because my predicted winner is likely to lose before the championship game.]

What is the expected value of my total score?

Solution:

I expect to score 31.5 points.

Proof. The tournament continues until all teams are eliminated. This means that exactly 63 games have to be lost — so the tournament is made up of exactly 63 games.

Let g be a game in the tournament, and define

$$X_g = \begin{cases} 1 & \text{if I score points for my prediction on game } g, \\ 0 & \text{if I don't.} \end{cases}$$

If game g happens in round n, then a correct prediction of game g is worth 2^{n-1} points. To earn those points, I have to predict n games correctly — namely, the first n games involving the winner of game g. Thus $P(X_g = 1) = 2^{-n}$, and the expected number of points I score on game g is $2^{n-1} \cdot 2^{-n} = \frac{1}{2}$.

By the linearity of expectation, my expected total score is $\sum_g \frac{1}{2} = \frac{63}{2} = 31.5$. □

Source: D. Barsky, via Janko Gravner and Michael Black:
`https://www.math.ucdavis.edu/~gravner/MAT135A/resources/chpr.pdf`

*The play-in games don't count.