UNIVERSITY OF WISCONSIN
PLATTEVILLE
DEPARTMENT OF MATHEMATICS

Problem of the Week #4

(Spring 2017)

An island has a population of chameleons. Currently, 34 of them are red, 21 are green, and 47 are blue. Whenever two chameleons of different colors meet, they both change to the third color. There are no color changes otherwise. Is it possible that at some future time, after a sequence of meetings, all of the chameleons will be the same color?

Solution:
The chameleons will never all be the same color.

Proof. Let r be the number of red chameleons and g be the number of green chameleons, and let $X = (r - g) \bmod 3$. Currently, $r - g = 13$, so $X = 1$.
If a red and a green chameleon meet and turn blue, then r goes down by 1 and g goes down by 1. So $r - g$ doesn't change, leaving $X = 1$.
If a red and a blue chameleon meet and turn green, then r goes down by 1 and g goes up by 2. So $r - g$ decreases by 3, leaving $X = 1$.
If a green and a blue chameleon meet and turn red, then r goes up by 2 and g goes down by 1. So $r - g$ increases by 3, leaving $X = 1$.
So $X = 1$ after any sequence of color changes.
But if all the chameleons were the same color, then we would have $r - g \in \{-102, 0, 102\}$, and therefore $X = 0$. Therefore, it is not possible for all the chameleons to turn the same color.
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Source: University of Michigan Undergraduate Mathematics Competition, 2014.
`http://lsa.umich.edu/math/undergraduates/extracurricular-activities/competitions/undergraduate-mathematics-competition.html`