University of Wisconsin Platteville
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Problem of the Week #8

(Fall 2023)

Arriving at the very heart of the Oblivion Labyrinth, four heroes discovered a treasure vault, heavily shielded both from magic and from trickery. Inside the vault, within an adamantine chest, they beheld the Thirteen Rings of Power. They all agreed that the fiendish potential of the Rings was too great for easily tempted mortals such as themselves, and left the vault for a long rest. Only much later did Lord Richmere discover that all thirteen Rings had been stolen.

It was well known that only the four heroes had ever penetrated the Labyrinth, so they certainly must have taken all of the Rings. Gazing back through time, Richmere's mages saw that each of the four had re-entered the vault just once, separately. Unable to pierce the vault's arcane veil of secrecy, the mages summoned the four heroes, lured them into a *zone of truth,* and extracted the following true confessions:

- Alchemist: "I stole an odd number of Rings."
- Barbarian: "I stole an even number of Rings."
- Cavalier: "I stole exactly two thirds of the Rings I found in the chest."
- Druid: "I stole exactly one quarter of the Rings I found in the chest."

Which hero stole the most Rings?

Solution:
The alchemist stole the most Rings.

Proof. The number of Rings found in the chest by the cavalier must be a multiple of 3. So the cavalier didn't find all 13 Rings, and therefore didn't enter first. On the other hand, the cavalier didn't take all 13 Rings, so didn't enter last. Similarly, the druid didn't enter first or last. The cavalier and the druid entered second and third, not necessarily in that order, and so the first to enter was either the alchemist or the barbarian.

If the barbarian entered first, they took an even number of Rings and left an odd number. Odd numbers aren't divisible by 4, so the cavalier entered next, found an odd number of Rings divisible by 3 — either 3 or 9 — and left either 1 or 3 Rings. That's impossible, though, since the druid must enter next, and 1 and 3 are not divisible by 4.

Thus the alchemist entered first and left behind n Rings. The cavalier and the druid, entering second and third in some order, left $n \cdot \frac{1}{3} \cdot \frac{3}{4} = \frac{n}{4}$ Rings for the barbarian to find, and the barbarian must have stolen them all. Thus $\frac{n}{4}$ is a non-negative even integer less than or equal to $\frac{13}{4} = 3.25$, so $n = 0$ or $n = 8$. Either the alchemist took all 13 Rings, or they took 5 and then the druid took 2, the cavalier took 4, and the barbarian took 2. □

Source: Tadao Kitazawa, *Arithmetical, Geometrical and Combinatorial Puzzles from Japan*, MAA Press (2021), 2–8.