UNIVERSITY OF WISCONSIN
PLATTEVILLE
DEPARTMENT OF MATHEMATICS

Problem of the Week #6

(Fall 2023)

Rick is on vacation on Trichotomy Island. The roads on this island have been laid out so that exactly three (two-way) roads come together at each intersection. That is, at any point where Rick has the opportunity to turn, he can either take a left fork or a right fork. (He can't go back the way he came, because U-turns aren't allowed.)

It's a nice day, and Rick wants to go for a drive in his convertible, but he doesn't know his way around, and he doesn't want to get lost, so he makes a plan. Once he leaves his parking place, he will turn right at the first intersection, then left at the next intersection, then right, then left, and so on. Prove that if Rick follows this plan, he will eventually get back to his original parking place.

Solution:

Proof. Whenever Rick is between intersections, the current status of his trip can be fully described by three pieces of information: the stretch of road he's on, the direction he's driving, and which way he's about to turn. Hence there are only finitely many possibilities for his status, and so if he drives long enough, he will eventually find himself in a status he's been in before.

The first time Rick repeats a status, he is on the road where he started his trip. Otherwise, he just turned off of a different stretch of road, and it must be the same stretch of road, from which he turned (the same direction) the first time he was in this status, so his previous status was also a repeat. □

Source: "Touring an Island." In: Peter Winkler, *Mathematical Mind-Benders*, A.K. Peters, Ltd. (2007), 66, 69.