



UNIVERSITY OF WISCONSIN  
**PLATTEVILLE**  
DEPARTMENT OF MATHEMATICS

PROBLEM OF THE WEEK #3  
(Fall 2023)

You're about to play a game. On each turn, you'll draw a number at random from a hat, then make a choice. If you draw the number  $k$ , then you may collect  $k$  dollars, or you may instead add to the hat the  $k$  smallest positive integers that aren't already in the hat. For example, if your turn begins with the numbers 1 through 8 in the hat, and you draw the number 3, you may either collect \$3 or add the numbers 9, 10, and 11 to the hat.

When the game starts, only the number 1 is in the hat. If you get to play for 100 turns, and you choose your strategy to maximize the amount of money you collect, on average how much money can you expect to win?

[Please fully explain your answer.]

Email solutions to [swensonj@uwplatt.edu](mailto:swensonj@uwplatt.edu) by 2:00pm on Wednesday, October 4, 2023.

Every week, the best solution submitted earns a \$10 Platteville gift certificate; the top scorer each semester also wins a cash award. Good luck!

You can always see the Problem of the Week (and complete rules) online at:

<http://uwpmath.weebly.com/>