University of Wisconsin Platteville
Department of Mathematics

Problem of the Week #1

(Fall 2023)

Let's shuffle a standard deck of 52 cards (no jokers), then reveal cards one by one until we've seen three kings. On average, how many cards do we expect we'll see?

Solution:

We should expect to see 31.8 cards.

Proof. If we keep going to the end of the deck, we'll reveal C_0 cards before the first king turns up, then C_1 cards between the first and second king, C_2 between the second and third, C_3 between the third and fourth, and C_4 after the fourth. Counting all non-kings in the deck, we get $C_0 + C_1 + C_2 + C_3 + C_4 = 52 - 4 = 48$.

Now take expected values. Because the cards are shuffled randomly, we know that all five values of $E(C_i)$ are equal. Then, by linearity of expectation,

$$E(C_i) = \frac{1}{5} \cdot 5E(C_i) = \frac{1}{5}\left[E(C_0) + \cdots + E(C_4)\right] = \frac{1}{5}E(C_0 + \cdots + C_4) = \frac{48}{5}.$$

This means that the expected number of cards revealed is

$$E(C_0 + 1 + C_1 + 1 + C_2 + 1) = 3 + 3 \cdot \frac{48}{5} = \boxed{\frac{159}{5}} = \boxed{31.8}.$$

□

Source: George Stoica, "Quickies 1119," *Mathematics Magazine* **95**:2 (April 2022), 158.