PROBLEM OF THE WEEK #9
(Fall 2022)

Given real numbers $a_0 > b_0 > 0$, construct a pair of sequences inductively as follows. For each n, let a_{n+1} be the arithmetic mean of a_n and b_n:

$$a_{n+1} = \frac{a_n + b_n}{2}.$$

And let b_{n+1} be the harmonic mean of a_n and b_n:

$$\frac{1}{b_{n+1}} = \frac{\frac{1}{a_n} + \frac{1}{b_n}}{2}.$$

Show that the sequences $\{a_n\}$ and $\{b_n\}$ both converge to the same limit, which is $\sqrt{a_0 b_0}$, the geometric mean of a_0 and b_0.

[Please fully explain your answer.]

Email solutions to `kwonmi@uwplatt.edu` by 4:00pm on Wednesday, November 16, 2022.

Every week, the best solution submitted earns a \$10 Platteville gift certificate; the top scorer each semester also wins a cash award. Good luck!
You can always see the Problem of the Week (and complete rules) online at:

`http://uwpmath.weebly.com/`