UNIVERSITY OF WISCONSIN
PLATTEVILLE
DEPARTMENT OF MATHEMATICS

Problem of the Week #6

(Fall 2022)

Notice that $2^2 + 2^2 + 3^2 = 17$, which is a prime number.
Suppose that a, b, and c are prime numbers, all of which are greater than or equal to five. Show that $a^2 + b^2 + c^2$ is not prime.

Solution:

Proof. Since a is prime and $a \geq 5$, a is not divisible by 3. Working modulo 3, we have $a \equiv 1$ or $a \equiv 2$, which means $a^2 \equiv 1^2 = 1$ or $a^2 \equiv 2^2 \equiv 1$. Likewise, $b^2 \equiv 1$ and $c^2 \equiv 1$, so $a^2 + b^2 + c^2 \equiv 1 + 1 + 1 \equiv 0$. That is, $a^2 + b^2 + c^2$ is a multiple of 3. But of course, $a^2 + b^2 + c^2 \geq 3 \cdot 5^2 = 75 > 3$. $\square$

Source: Ross Honsberger. *Mathematical Gems II.* The Dolciani Mathematical Expositions, No. 2. The Mathematical Association of America (1976), 37.