UNIVERSITY OF WISCONSIN
PLATTEVILLE
DEPARTMENT OF MATHEMATICS

Problem of the Week #2

(Fall 2022)

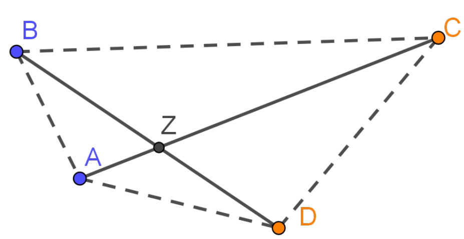
Suppose we are given n blue points and n orange points in the plane, selected so that no three of the $2n$ points lie on a single line.
Prove that each of the blue points can be given its own orange partner in such a way that the line segments joining points to their partners do not cross.

Solution:

Proof. Given any pairing, let A and B be blue points, and let C be the orange point paired with A and D the orange point paired with B. If segment $\overline{AC}$ intersects $\overline{BD}$ at Z, then by the triangle inequality,

$$AD + BC < AZ + ZD + BZ + ZC = AC + BD,$$

so pairing A with D and B with C would give a new pairing in which the lengths of the partner-segments add up to a lower total.
There are only finitely many possible pairings, so some pairing has the minimum total partner-segment length, and that pairing cannot have intersecting segments. □



Source: Peter Winkler, "Red Points and Blue Points," *Mathematical Puzzles: A Connoisseur's Collection,* A.K. Peters, Ltd. (2004), pp. 45, 49.