



PROBLEM OF THE WEEK #1  
(Fall 2022)

This week, across the United States, the 48 contiguous states are choosing sides for a very big game of capture-the-flag. To make things work right, we need 24 states on the orange team and 24 on the blue team, and each team has to be contiguous — the 24 states have to form a single connected region.

It's time for the orange team to make its fourth pick: so far, they've chosen California, Colorado, and New Mexico, while the blue team has Florida, Missouri, and Texas. The orange team wants to select Virginia. Is that legal? In other words, can the draft then be completed so that both teams comprise 24 contiguous states?

**Note:** Michigan is a little complicated, because it's not all one piece. Let's agree to count Michigan as a single piece anyway, with Wisconsin, Indiana, and Ohio as its neighbors.

**Solution:**

It is not legal for the orange team to select Virginia.

*Proof.* Assume given a choice of teams with 24 contiguous orange states and 24 contiguous blue states, in which California, Colorado, and New Mexico are orange, but Texas is blue. Suppose for contradiction that Virginia is orange.

The contiguity of the blue team means that any two blue states must be connected by a sequence of blue states, each bordering the next (a "blue path"), and similarly for orange states.

Suppose for contradiction that Wisconsin is blue. Then there is a blue path from Wisconsin to Texas, so there's no orange path from California to Virginia, contrary to hypothesis. Thus Wisconsin is orange. By the same argument, all border states from California clockwise to Virginia are orange. That's California, Oregon, Washington, Idaho, Montana, North Dakota, Minnesota, Wisconsin, Michigan, Ohio, Pennsylvania, Vermont, New Hampshire, New York, Maine, Massachusetts, Rhode Island, Connecticut, New Jersey, Delaware, Maryland, and Virginia. Remember that Colorado and New Mexico are also orange.

That's already 24 orange states, so the rest of the states are blue, which means the orange states aren't contiguous: Colorado and New Mexico are disconnected from the other orange states.

From this contradiction, we conclude that Virginia can't be orange. □

**Source:** Ivor Traber, "Riddler Classic,"

<https://fivethirtyeight.com/features/can-you-crack-and-pack-riddler-nation/>

**Map:** <https://www.gasfoodnolodging.com/visitedstates/generate/>