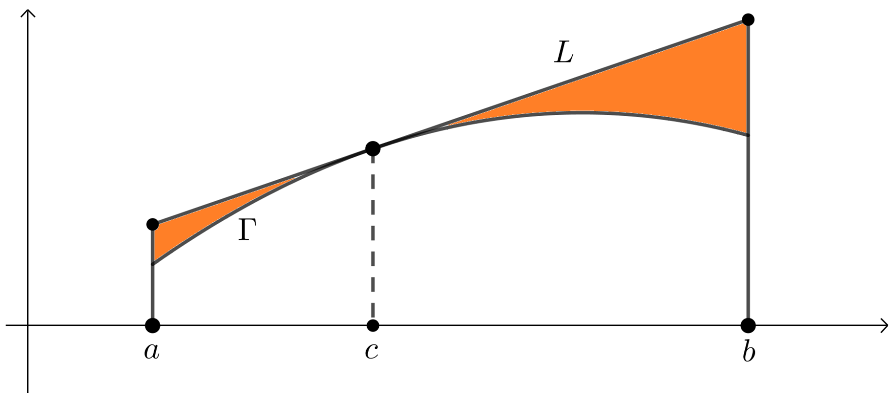
UNIVERSITY OF WISCONSIN
PLATTEVILLE
DEPARTMENT OF MATHEMATICS

Problem of the Week #10

(Fall 2021)

Let Γ be the graph of a function that is differentiable and concave down on the interval $[a, b]$, and let L be a line tangent to Γ at $x = c$.
What value of c minimizes the shaded area, which is bounded by Γ, L, $x = a$, and $x = b$?



Solution:

The area is minimized when $c = \frac{1}{2}(a + b)$, at the midpoint of $[a, b]$.

Proof. The area between Γ and the x-axis is constant, so it is enough to minimize the area of the trapezoid above $[a, b]$ and below L. That means we have to minimize the average height of the trapezoid, which is the height of L at $x = \frac{1}{2}(a + b)$. To accomplish this, we just have to make sure that L and Γ have the same height at $x = \frac{1}{2}(a + b)$, and to do this we must set $c = \frac{1}{2}(a + b)$. □

Source: Robert Paré. "A Visual Proof of Eddy and Fritsch's Minimal Area Property." *College Mathematics Journal* **26**:1 (January 1995), 43–44.
Cited in: Claudi Alsina and Roger B. Nelson. *Charming Proofs: A Journey Into Elegant Mathematics.* The Mathematical Association of America (2010), 158, 265.