



PROBLEM OF THE WEEK #7  
(Fall 2021)

An eccentric mathematician gives you 25 pennies, and invites you to divide them into piles. Once you do, she will count the pennies in each pile, multiply together all of the pile sizes, and pay you a number of pennies equal to the product. What is the largest payout you can earn?

**Solution:**

You can earn (at most) \$87.48.

*Proof.* If you have a pile of  $n$  pennies and a pile with only 1 penny, it's better to combine them, because  $n + 1 > n \cdot 1$ . So you shouldn't make any piles with only one penny.

You also shouldn't make any piles with 5 or more pennies, because, if  $n \geq 5$ , we have  $2(n - 2) = 2n - 4 > 2n - 5 \geq n$ , which means it's better to split a pile of  $n$  pennies into a pile of 2 and a pile of  $n - 2$ . [Of course, some other option might be even better.]

In fact, you might as well split a pile of 4 pennies into two piles of 2, because  $2 \cdot 2 = 4$ . So you can earn your maximum payout using only piles of two or three pennies.

You shouldn't have more than two piles of 2 pennies, because  $2^3 = 8 < 9 = 3^2$ , so three piles of 2 should always be turned into two piles of 3.

So your best option is to make seven piles of 3 and two piles of 2, for a total payout of  $3^7 \cdot 2^2 = 8748$  cents.  $\square$

**Source:** Shapiro, Austin. "Pen and Paper Math 6 Solutions." <http://dekapus.com/pp6.pdf>