



PROBLEM OF THE WEEK #5
(Fall 2021)

In a 9630×9630 square grid, each 1×1 square is either blue or orange. Each blue square that isn't on the border has exactly four orange neighbors (either horizontally, vertically, or diagonally), and each orange square that isn't on the border has exactly five blue neighbors. How many squares are orange?

Solution:

There are exactly 41,216,400 orange squares.

Proof. Let's use the word "block" for a 3×3 portion of the grid. The grid may be divided into $3210^2 = 10,304,100$ non-overlapping blocks. A block with an orange square in the center contains exactly 5 blue squares and therefore 4 orange squares. A block with a blue square in the center also contains exactly 4 orange squares. So the grid contains exactly $4(10,304,100) = 41,216,400$ orange squares. \square

Source: Velleman, Daniel J., and Stan Wagon. "A Black and White Issue." *Bicycle or Unicycle?* Providence: MAA Press (2020), 22, 138.