University of Wisconsin
PLATTEVILLE
Department of Mathematics

Problem of the Week #1

(Fall 2020)

Find an integer n for which the first four digits of n^{100000} are all distinct.

Solution:

There are many, many solutions, but it's probably easiest to think of $n = 100001$.

Proof. This solution is based on the fact that $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$.

$$100001^{100000} = \left(\frac{100001}{100000}\right)^{100000}\left(100000^{100000}\right) = \left(1.00001^{100000}\right)\left(10^{500000}\right),$$

which has the same significant digits as $1.00001^{100000} = 2.718268\ldots$.
So the first four significant digits of 100001^{100000} are 2718: four distinct digits. $\square$

Source: Stanley, Richard P. *Combinatorial Problem Solving,* American Mathematical Society (2020), 132.