



PROBLEM OF THE WEEK #7
(Fall 2019)

In a desperate effort to provide a moment of respite amid our panicked flight from the fiends that pursued us, I checked my dear companions into a secure facility, insisting urgently that they stay alert and stay together. Unfortunately, the facility was so secure that when I returned at last, having laid enough false trails to deceive the monsters, I couldn't find my friends. They were nowhere to be seen, and in this particular institution, only the most naïve visitor would open a door without knowing what might be lurking on the other side.

The three guards maintained a disciplined silence in each other's presence, but when I got them alone, they were willing enough to talk. "They're either in cell 6B or 8C," the first guard told me. I checked with the second guard, who said, "They're either in cell 8C or 1A." This seemed to clear it up, but I sought out the third guard anyway. "They're either in cell 1A or 6B," she told me. "But be careful! There are two vampires here acting as guards."

I was stunned, and not only by the revelation that I had led my friends (and myself) into grave peril. I had been relying on the well-known fact that guards always tell the truth — except for vampire guards, who always lie. How could I find my companions now?

Solution:

The companions are in cell 8C.

Proof. Suppose for the sake of contradiction that the third guard is not a vampire. Then it is true that the other two guards are vampires. It follows that the friends are not being kept in cells 1A, 6B, or 8C. But this means that the third guard's first sentence is false, which is impossible.

This shows that the third guard is a vampire. Because she always lies, there are not two vampire guards, which implies that the first two guards are not vampires. They both told the truth, so the companions are waiting in cell 8C. ♣

Source: Suggested by: Barry R. Clarke. "The Three Doors." *Challenging Logic Puzzles*. New York: Puzzle Wright Press (2003), pp. 14, 59.