University of Wisconsin
PLATTEVILLE
Department of Mathematics

Problem of the Week #6

(Fall 2019)

Let a, b, and c be the lengths of the sides of a triangle. Suppose that

$$a^2 + b^2 + c^2 = ab + bc + ca.$$

Prove that the triangle is equilateral.

Solution:

$$\begin{aligned}
a^2 + b^2 + c^2 &= ab + bc + ca \\
2a^2 + 2b^2 + 2c^2 &= 2ab + 2bc + 2ca \\
a^2 + b^2 + b^2 + c^2 + c^2 + a^2 &= 2ab + 2bc + 2ca \\
a^2 - 2ab + b^2 + b^2 - 2bc + c^2 + c^2 - 2ca + a^2 &= 0 \\
(a-b)^2 + (b-c)^2 + (c-a)^2 &= 0
\end{aligned}$$

This sum of three non-negative real numbers equals zero, so each term must equal zero. Thus $a = b$, $b = c$, and $c = a$, so the triangle is equilateral.

Source: M. S. Klamkin, *Mathematics Magazine* **27** (May 1954), 287. Quoted in: "The Triangle Is Equilateral." Charles W. Trigg, *Mathematical Quickies: 270 Stimulating Problems with Solutions*. New York: Dover Publishing Inc. (1985), p. 14, 95.