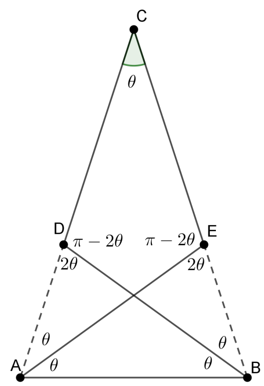
UNIVERSITY OF WISCONSIN
PLATTEVILLE
DEPARTMENT OF MATHEMATICS

Problem of the Week #1

(Fall 2019)

Five matchsticks of equal length lie end to end, with their ends on the sides of a triangle, as shown in the figure below. What is the measure of the marked angle at the top vertex of the triangle?



Solution:

The angle measure is $\frac{\pi}{5}$ radians, or $36°$.

Proof. Label points as in the figure, and let $\theta = \angle ACB$. Since $\triangle ACE$ and $\triangle BCD$ are isosceles, $\angle CAE = \angle CBD = \theta$, which leaves $\angle AEC = \angle BDC = \pi - 2\theta$. Hence $\angle ADB = \angle AEB = 2\theta$. Since $\triangle ABD$ and $\triangle ABE$ are isosceles, $\angle BAD = \angle ABE = 2\theta$, and so $\angle BAE = \angle ABD = \theta$. Now, from $\triangle ABD$ (or $\triangle ABE$), we have $5\theta = \pi$, so $\theta = \frac{\pi}{5}$. $\square$

Source: "Five Matches." Alex van den Brandhof, *et al.* *Half a Century of Pythagoras Magazine*. MAA Press (2015), p. 231.