University of Wisconsin
PLATTEVILLE
DEPARTMENT OF MATHEMATICS

Problem of the Week #9

(Fall 2018)

Write as small a positive real number as you can. The smallest positive real number wins! You may use any standard mathematical notation, but *you may not use more than five characters.* The grader is the final judge of how many characters you used. Ambiguous answers will be disqualified.

Examples:

- "$\frac{1}{999}$" is written with exactly five characters, counting the fraction bar.
- "0.001" is written with exactly five characters, counting the decimal point, and is less than $\frac{1}{999}$.
- ".0001" is written with exactly five characters, counting the decimal point, and is less than 0.001.
- "$\sin 710$" is disqualified for using six characters, counting sin as three.
- "$\ln \ln 2$" is disqualified for not being positive.
- "$\hbar$" is disqualified for not being standard *mathematical* notation.
- "ε" is disqualified for being ambiguous.
- "$0.\bar{0}1$" is disqualified for not being a real number.

Solution:

We received a, b, c, and d (below) as entries. Using Stirling's approximation, we get:

$a = 9^{-9^{9^9}}$	$b = 9^{-9^{99}}$	$c = 9!^{-9!}$	$d = \frac{1}{99!}$
$1/a = 9^{9^{9^9}}$	$1/b = 9^{9^{99}}$	$1/c = 9!^{9!}$	$1/d = 99!$
$\log_9(1/a) = 9^{9^9}$	$\log_9(1/b) = 9^{99}$	$\log_9(1/c) \approx 9! \cdot 9 \left(1 - \frac{1}{\ln 9}\right)$	$\log_9(1/d) \approx 99 \log_9 99 \left(1 - \frac{1}{\ln 99}\right)$
$\log_9 \log_9(1/a) = 9^9$	$\log_9 \log_9(1/b) = 99$	$\log_9 \log_9(1/c) \approx 6.55$	$\log_9 \log_9(1/d) \approx 2.43$

Thus $0 < a < b < c < d$.