UNIVERSITY OF WISCONSIN
PLATTEVILLE
DEPARTMENT OF MATHEMATICS

Problem of the Week #8

(Fall 2018)

A *palindrome* is a natural number that doesn't change when the order of its digits is reversed. For example:

- 168861 is a palindrome in base ten.
- $51 = 110011_2$ is a binary palindrome, but is not a palindrome in base ten.
- 0 is a palindrome in every base.

Write 2018 as a sum of three palindromes, and then write $2018 = 11111100010_2$ as a sum of three binary palindromes.

Solution:
By repeatedly choosing the greatest palindrome less than the desired sum, we can discover that $2018 = 2002 + 11 + 5$. (This procedure is called "the greedy algorithm.")
In binary, the same trick works, and we only need two positive palindromes: we have $11111011111_2 + 11_2 + 0_2 = 11111100010_2$. (In base ten, this says $2015 + 3 + 0 = 2018$.)

Remark.

- Every natural number can be written as a sum of three palindromes in any base $b \geq 5$, but it cannot always be done using the greedy algorithm.
- Every natural number can be written as a sum of four binary palindromes. The smallest natural number that cannot be written as a sum of three binary palindromes is $176 = 10110000_2$.

Source:

[CLB18] Javier Cilleruelo, Florian Luca, and Lewis Baxter, *Every positive integer is a sum of three palindromes*, Math. Comp. **87** (2018), no. 314, 3023–3055, DOI 10.1090/mcom/3221. MR3834696

[RSS18] Aayush Rajasekaran, Jeffrey Shallit, and Tim Smith, *Sums of palindromes: an approach via automata*, 35th Symposium on Theoretical Aspects of Computer Science, LIPIcs. Leibniz Int. Proc. Inform., vol. 96, Schloss Dagstuhl. Leibniz-Zent. Inform., Wadern, 2018, pp. Art. No. 54, 12. MR3779335