UNIVERSITY OF WISCONSIN PLATTEVILLE
DEPARTMENT OF MATHEMATICS

PROBLEM OF THE WEEK #4
(Fall 2018)

Simplify:

$$\frac{\dfrac{\frac{1}{x^2}+\frac{1}{y^2}}{\frac{1}{x^2}-\frac{1}{y^2}}-\dfrac{\frac{1}{x^2}-\frac{1}{y^2}}{\frac{1}{x^2}+\frac{1}{y^2}}}{\left(\dfrac{8}{\left(\frac{x+y}{x-y}+\frac{x-y}{x+y}\right)\left(\frac{x^2}{y^2}+\frac{y^2}{x^2}-2\right)}\right)}$$

Solution:

$$= \frac{\dfrac{y^2+x^2}{y^2-x^2}-\dfrac{y^2-x^2}{y^2+x^2}}{\left(\dfrac{8}{\left(\frac{2(x^2+y^2)}{x^2-y^2}\right)\left(\frac{x^4-2x^2y^2+y^4}{x^2y^2}\right)}\right)}$$

$$= \frac{\left(\dfrac{4x^2y^2}{y^4-x^4}\right)}{\left(\dfrac{4x^2y^2(x^2-y^2)}{(x^2+y^2)(x^2-y^2)^2}\right)}$$

$$= \frac{\left(\dfrac{1}{y^4-x^4}\right)}{\left(\dfrac{1}{(x^2+y^2)(x^2-y^2)}\right)}$$

$$= \frac{x^4-y^4}{y^4-x^4}$$

$$= \boxed{-1}.$$

Source:

[Chr86] George Chrystal, *Algebra: An Elementary Text Book for the Higher Classes of Secondary Schools and for Colleges*, A. and C. Black, 1886.

[Dud10] Underwood Dudley, *What is mathematics for?*, Notices Amer. Math. Soc. **57** (2010), no. 5, 608–613.