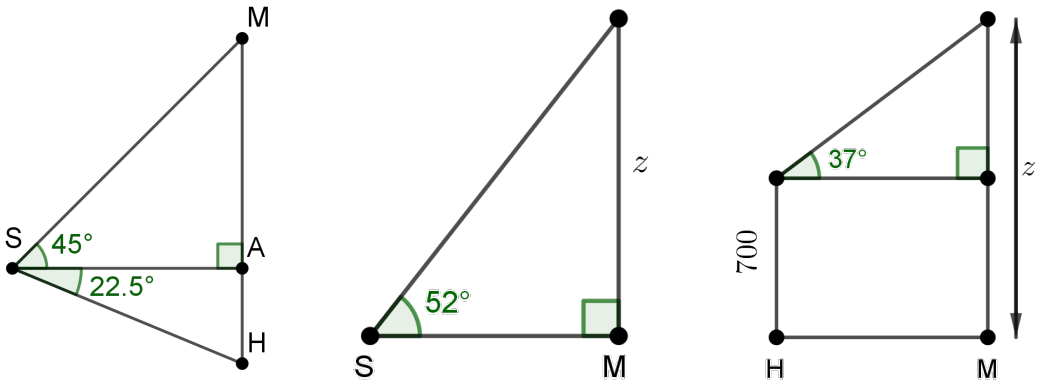
UNIVERSITY OF WISCONSIN
PLATTEVILLE
DEPARTMENT OF MATHEMATICS

Problem of the Week #3

(Fall 2018)

I work at a sea-level observation station. There's a mountain northeast of my station, and to the east-southeast, there's a 700 ft hill.
From my station, the mountain peak is at an elevation of 52° above the horizon. From the top of the hill, the mountain peak is due north, at a 37° elevation.
How high is the mountain?

Solution:
We shall not correct for the curvature of the earth.
Let the observation station be located at S in the plane, the mountain peak above M, and the hilltop above H. The perpendicular through S meets HM at A; let z denote the height of the mountain.



Both $\angle MSH$ and $\angle MHS$ measure 67.5°, so $\triangle SMH$ is isosceles, and therefore

$$\begin{aligned} HM &= SM \\ (z-700)\cot 37^\circ &= z\cot 52^\circ \\ z(\cot 37^\circ - \cot 52^\circ) &= 700\cot 37^\circ \\ z &= \boxed{700\left(\frac{\cot 37^\circ}{\cot 37^\circ - \cot 52^\circ}\right)} \\ z &\approx 1702\,\text{ft} \end{aligned}$$

Source: John Hopkins, based on [Per55]:

[Per55] I. E. Perlin, *Trigonometry*, International Textbook Company, Scranton, 1955.