UNIVERSITY OF WISCONSIN
PLATTEVILLE
DEPARTMENT OF MATHEMATICS

Problem of the Week #10

(Fall 2017)

Some 1s and some −1s are arranged in a circle: 2017 of each, in fact. Let S be the sum of all 4034 products of adjacent pairs of numbers around the circle. Prove that no matter how the 1s and −1s are arranged, $S \neq 0$.

Solution:
Suppose for the sake of contradiction that $S = 0$. Since each product in the sum equals ± 1, there must be exactly 2017 positive products and 2017 negative products. That means that if we walk once around the circle, we find exactly 2017 sign changes (where a 1 is adjacent to a −1). But we end at the same number where we started, so the number of sign changes must be even. $\Rightarrow\Leftarrow$.

Source: "Zero-Nonzero." *Math Horizons* **25**:2 (November 2017), pp. 32-33.