



PROBLEM OF THE WEEK #7
(Fall 2017)

Some of the citizens of Transylvania are human; the others are vampires. The humans always try to tell the truth, and the vampires always try to lie. Unfortunately, some of the inhabitants are insane: when a claim is true, the insane citizens believe it to be false, and *vice versa*. The sane Transylvanians, on the other hand, are completely accurate: they always know which claims are true and which are false.


I was talking with Boris and Natasha, a married Transylvanian couple. [As you know, a human who marries a vampire always becomes a vampire.] “We are vampires,” Boris said. “Yes, we are,” agreed Natasha. “As far as sanity goes,” said Boris, “we’re also alike.” What can you deduce about Boris and Natasha?

Solution:

Boris and Natasha are insane vampires.

Proof. Any given person is a truth-teller (sane human or insane vampire) or a liar (insane human or sane vampire). Boris said the same thing Natasha said, so they’re either both truth-tellers or both liars.

Suppose they’re liars. Then (because Boris lied) one of them is sane and the other is insane; specifically, because they’re liars, one is a sane vampire and the other is an insane human. But a human can’t be married to a vampire (humans who marry vampires become vampires), so this is impossible.

Thus both are truth-tellers. Hence we can trust Boris, so both are vampires, which means both are insane. 

Source: Smullyan, Raymond. *The Lady or the Tiger? and other logic puzzles*. New York: Alfred A. Knopf (1982), pp. 50, 56.