PROBLEM OF THE WEEK #3

(Fall 2017)

The digits 0, 1, 2, ..., 9 (one copy of each) have fallen out of the number

$$x = 5\_383\_8\_2\_936\_5\_8\_203\_9\_3\_76$$

Based on this information, what is the probability that x is divisible by 396?

Solution:
There is some integer n for which $x = 100n + 76 = 4(25n + 19)$, so x is divisible by 4.
The sum of the digits of x is $90 + 45 = 135 = 9 \cdot 15$. Since this is divisible by 9, so is x.
The alternating sum of the digits of x is $\pm(73 - (17 + 45)) = 11$. Since this is divisible by 11, so is x.
Since 4, 9, and 11 are relatively prime, x is divisible by $4 \cdot 9 \cdot 11 = 396$ (with probability 1).

Source: Na Nagara, Prasert. *American Mathematical Monthly* 58 (December 1951), p. 700. Quoted in Trigg, Charles W. "Divisibility Probability." *Mathematical Quickies: 270 Stimulating Problems with Solutions.* New York: Dover Publications, Inc. (1985), pp. 18, 101.