UNIVERSITY OF WISCONSIN
PLATTEVILLE
DEPARTMENT OF MATHEMATICS

Problem of the Week #10

(Fall 2016)

In my new favorite game, I roll n fair 6-sided dice, and I win if every number from 1 to 6 comes up at least once. What is the smallest value of n for which I'm more likely to win than to lose?

Solution:
The smallest such value is $n = 13$.

Proof. Let $S = \{1, 2, 3, 4, 5, 6\}$, and let $A \subseteq S$. Under the assumption that the n die rolls are independent, the probability that every die shows an element of A is $\left(\frac{|A|}{6}\right)^n$.
Therefore, by the inclusion-exclusion principle, the probability that I win is

$$\sum_{A \subseteq S} (-1)^{|A|} \left(\frac{|A|}{6}\right)^n = \sum_{k=0}^{6} (-1)^k \binom{6}{k} \left(\frac{k}{6}\right)^n.$$

With electronic assistance, we can show that this probability is about 0.438 when $n = 12$, and about 0.514 when $n = 13$. □

Source: Prof. Michael Black.