



PROBLEM OF THE WEEK #6
(Fall 2016)

Given three distinct real numbers p , q , and r , find a quadratic polynomial $f(x)$ such that

$$\begin{cases} f(p) = q, \\ f(q) = r, \\ f(r) = p. \end{cases}$$

Solution:

$$f(x) = \frac{(x-p)(x-q)}{(r-p)(r-q)} p + \frac{(x-q)(x-r)}{(p-q)(p-r)} q + \frac{(x-r)(x-p)}{(q-r)(q-p)} r.$$

Remark. If you really want an answer in standard form:

$$f(x) = \frac{p^2 + q^2 + r^2 - pq - pr - qr}{(p-q)(p-r)(q-r)} x^2 + \frac{p^2r + q^2p + r^2q - p^3 - q^3 - r^3}{(p-q)(p-r)(q-r)} x + \frac{p^3q + q^3r + r^3p - p^2q^2 - p^2r^2 - q^2r^2}{(p-q)(p-r)(q-r)}.$$

Source:

Warren, Henry S. *Hacker's Delight*. Upper Saddle River, NJ: Addison-Wesley, 2013 (p. 48).